

# SPECTRAL THEORY FOR OPERATORS AND SEMIGROUPS

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## MAIN BODY

### LECTURE 1: A survey on Operator theory

- 1.1: Main definitions and properties of bounded operators on a Banach space. Dual operators and some basic properties.
- 1.2: Closed operators. Definitions and different characterizations. Closable operators and closure of an operator.
- 1.3: Spectrum and resolvent of a (bounded or closed) operator. Basic properties. Spectral radius.
- 1.4: Normal and selfadjoint operators and their basic properties.
- 1.5: Exercises.

### LECTURE 2: Compact operators and Riesz-Schauder theory

- 2.1: Compact operators: definitions, examples, Schauder theorem.
- 2.2: Riesz-Schauder theory for compact operators.
- 2.2: Spectral decomposition theorem for selfadjoint compact operators.
- 2.3: Exercises.

### LECTURE 3: Spectral representation theorem for bounded operators

- 3.1: Spectral representation theorem for bounded and selfadjoint operators on a separable Hilbert space  $H$ .
- 3.2: Spectral theorem for normal operators.
- 3.3: Exercises.

### LECTURE 4: Spectral representation theorem for unbounded operators

I

- 4.1.: Adjoint of an unbounded operator, selfadjoint and symmetric operators. Definitions and basic properties.
- 4.2.: Dissipative operators. Definitions and main properties.
- 4.3.: Representation theorem for unbounded selfadjoint operators.
- 4.4.: Spectral mapping theorem for the resolvent operator of closed linear operator.
- 4.5: Exercises.

### LECTURE 5: Spectral representation theorem for unbounded operators

II

- 5.1: Spectral representation theorem for selfadjoint operators.
- 5.2: Positive operators and minimax theorems for their eigenvalues.
- 5.3: Exercises.

**LECTURE 6: Strongly continuous semigroups**

- 6.1: Definitions and basic properties.
- 6.2: The infinitesimal generator.
- 6.3: Hille-Yosida theorem.
- 6.4: Exercises.

**LECTURE 7: Analytic semigroups**

- 7.1: Sectorial operators and analytical semigroups.
- 7.2: Main properties of analytical semigroups.
- 7.3: Exercises.

**LECTURE 8: Some relevant examples of semigroups**

- 8.1: The multiplication semigroup.
- 8.2: The semigroups of translations.
- 8.3: The heat semigroup.
- 8.4: Exercises.

**LECTURE 9: Abstract Cauchy problems I**

- 9.1: Strong, mild, classical and strict solutions.
- 9.2: Abstract Cauchy problems associated with strongly continuous semigroups.
- 9.3: Interpolation spaces.
- 9.4: Exercises.

**LECTURE 10: Abstract Cauchy problems II**

- 10.1: Abstract Cauchy problems associated with analytic semigroups: classical and strict solutions.
- 10.2: Optimal time regularity results.
- 10.3: Optimal space regularity results.
- 10.4: Applications to Schauder estimates.
- 10.5: Exercises.

**LECTURE 11: Spectral mapping theorems for semigroups**

- 11.1: The spectral mapping theorem for the point and residual spectrum.
- 11.2: Spectral mapping theorem for eventually norm continuous semigroups and consequences.
- 11.3: Examples and counterexamples.
- 11.4: Exercises.

**LECTURE 12: Asymptotic behaviour of strongly continuous and analytic semigroups**

- 12.1: Stability and exponential stability.
- 12.2: Exponential dichotomy.
- 12.3: Exercises.

**LECTURE 13: Long time behaviour of solutions to nonhomogeneous abstract Cauchy problems**

- 13.1: Forward solutions.

- 13.2:** Backward solutions.
- 13.3:** Solutions defined on the whole line.
- 13.4:** Exercises.

#### **LECTURE 14: Some classes of abstract nonlinear problems**

- 14.1:** Nonlinear problems with nonlinearity defined in some intermediate space.
- 14.2:** The principle of linearized stability.
- 14.3:** A concrete example.
- 14.4:** Exercises.

### **APPENDICES**

#### **APPENDIX A: Tools**

- A.1:** Stone-Weierstrass theorem.
- A.2:** Riesz representation theorem.

#### **APPENDIX B: Differential and integral calculus for functions with values in a Banach space**

- B.1:** Differential calculus for functions defined on an interval with values in a Banach space: definitions and main properties.
- B.2:** Vector-valued Riemann integral: definitions and main properties.

#### **APPENDIX C: Holomorphic functions with values in a Banach space**

- C.1:** Holomorphic functions with values in a Banach space: definitions and main properties.
- C.2:** Taylor and Laurent expansions.